Numerical Methods Practical Warks
$2^{\text {nd }}$ year of a bachelor's degree in computer science

## Practical Work "03"

## MATLAB Exercises

## Exercise DI

「reate a vector that goes at equal steps from -2 to +2 containing 50 components.

## Exercise 02

1. Using the help command, find what daes the reshape command do?
2. Create the following matrix using one MATLAB line of code and the reshape command.

$$
M=\left(\begin{array}{ccc}
1 & 9 & 17 \\
3 & 11 & 19 \\
5 & 13 & 21 \\
7 & 15 & 23
\end{array}\right)
$$

## Exercise 03

Find and display in a vector all integers between I and IOUC which divide by 37. Propose at least two different ways to solve this problem.

## Exercise 04

Fibonacci numbers form a sequence starting with $\square$ followed by I. Each subsequent number is the sum of the previous two. Hence the sequence starts as $\mathrm{Z}, \mathrm{I}, \mathrm{I}, 2,3,5,8,13, \ldots$....
Write a non-recursive MATLAB function to calculate the Fibonacci sequence and return the number with a specified index.

## Exercise 05

Write a recursive MATLAB function to calculate the Fibonacci sequence and return the number with a specified index.

## Exercise [E

Write a MATLAB program (function) to calculate the trace of a square matrix A of order п п $\square$.
The trace of a matrix is given by: $\operatorname{tr}(A)=a_{11}+a_{22}+\cdots+a_{n n}$.

## Exercise 07

Write a MATLAB program to calculate the determinant of a matrix A of order $n>$ D.
The determinant of a matrix is given by: $\operatorname{det}(A)=\sum_{j=1}^{n}(-1)^{i+j} \times a_{i j} \times \operatorname{det}\left(A_{i j}\right)$.

## Exercise 08

Write a MATLAB program that allows you to verify the existence of the inverse of a matrix A then calculate this inverse if it exists.
The inverse of a matrix is given by: $A^{-1}=\frac{1}{\operatorname{det}(A)} \times C^{T}$
With $C=(-1)^{i+j} A_{i j}$ and $A_{i j}$ is the determinant of the matrix of size $(n-1) \times(n-1)$ obtained from A by deleting the $i^{\text {th }}$ row and $j^{\text {th }}$ column of $A$.
Use the previous function to calculate the determinant.

## Exercise 79

Write a MATLAB function that calculates the $\|v\|_{1}$ and $\|v\|_{2}$ and $\|v\|_{\infty}$ norms of a given vector $v$. Where: $\|v\|_{1}=\sum_{i=1}^{n}\left|v_{i}\right| . \quad\|v\|_{2}=\left(\sum_{i=1}^{n}\left|v_{i}\right|^{2}\right)^{\frac{1}{2}} . \quad\|v\|_{\infty}=\max _{i}\left|v_{i}\right|$

## Exercise 10

Write a MATLAB program that calculates the $\|A\|_{1}$ and $\|A\|_{F}$ and $\|A\|_{\infty}$ norms of a given matrix $A$. Where: $\|A\|_{1}=\max _{j} \sum_{i}\left|a_{i j}\right|$.

$$
\|A\|_{\infty}=\max _{i} \sum_{j}\left|a_{i j}\right| .
$$

$$
\|A\|_{F}=\sqrt{\operatorname{tr}\left(A^{T} A\right)}
$$

Use the trace function from exercise 6 .

## Exercise II

Write a MATLAB program to solve a system of linear equations $A x=b$ using Cramer's method. The solution is given by the following formula: $x_{i}=\operatorname{det} A_{i} / \operatorname{det} A$ with $A_{i}$ is a matrix obtained from $A$ by replacing the $i^{\text {th }}$ column of $A$ with the vector $b$.

## Exercise 12

Write a MATLAB program to solve a system of linear equations $A x=b$ using Gauss's elimination. The solution is given by:

$$
\left\{\begin{array}{c}
L_{1}^{(2)} \leftarrow L_{1}^{(1)} \\
\text { At first step }\left\{\begin{array}{c} 
\\
L_{i}^{(2)} \leftarrow L_{i}^{(1)}-\alpha_{i 1} L_{1}^{(1)} \text { où } \alpha_{i 1}=\frac{a_{i 1}^{(1)}}{a_{11}^{(1)}}
\end{array}\right. \\
\text { At } k^{\text {th }} \text { step }\left\{\begin{array}{c}
L_{i}^{(k+1)} \leftarrow L_{i}^{(k)}, 1 \leq i \leq k . \\
L_{i}^{(k+1)} \leftarrow L_{i}^{(k)}-\alpha_{i k} L_{k}^{(k)} \text { où } \alpha_{i k}=\frac{a_{i k}^{(k)}}{a_{k k}^{(k)}}, k+1 \leq i \leq n .
\end{array}\right.
\end{array}\right.
$$

## Exercise I3

Write a MATLAB program to solve a system of linear equations $A x=b$ using LU factorization.
The solution is given by:

$$
\left\{\begin{array}{c}
l_{i i}=1,1 \leq i \leq n ; \\
\text { first step }\left\{\begin{array}{l}
u_{1 j}=a_{1 j}, 1 \leq j \leq n ; \\
l_{i 1}=\frac{a_{i 1}}{a_{11}}, 2 \leq i \leq n ;
\end{array}\right. \\
k^{\text {th }} \operatorname{step}\left\{\begin{array}{c}
u_{m j}=a_{m j}-\sum_{k=1}^{m-1} l_{m k} \cdot u_{k j}, m \leq j \leq n \text { and } 2 \leq m \leq n \\
l_{i m}=\frac{\left(a_{i m}-\sum_{k=1}^{m-1} l_{i k} \cdot u_{k m}\right)}{u_{m m}}, m+1 \leq i \leq n \text { and } 2 \leq m \leq n
\end{array}\right.
\end{array}\right.
$$

